Mark Scheme (Results)

Summer 2024

Pearson Edexcel GCE
In A Level Further Mathematics (9FM0)
Paper 3D Further Decision 1

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL GCE MATHEMATICS
General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol $\surd$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- $*$ The answer is printed on the paper
- ⌐ The second mark is dependent on gaining the first mark

4. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

5. Where a candidate has made multiple responses and indicates which response they wish to submit, examiners should mark this response.
   If there are several attempts at a question which have not been crossed out, examiners should mark the final answer which is the answer that is the most complete.

6. Ignore wrong working or incorrect statements following a correct answer.

7. Mark schemes will firstly show the solution judged to be the most common response expected from candidates. Where appropriate, alternatives answers are provided in the notes. If examiners are not sure if an answer is acceptable, they will check the mark scheme to see if an alternative answer is given for the method used.

<table>
<tr><th>Qn</th><th>Scheme</th><th>Marks</th><th>AOs</th></tr>
<tr><td>1.(a)</td><td>The 16 was not placed in Bin 1 indicating that $n < 41$ (= 17 + 8 + 16), or the 4 was not placed in Bin 1 indicating that $n < 41$ (17 + 8 + 12 + 4)
Bin 2 contains 40 (= 16 + 24) so therefore $n = 40$</td><td>B1</td><td>2.4</td></tr>
<tr><td></td><td></td><td>(1)</td><td></td></tr>
<tr><td>(b)</td><td>e.g. Middle Right
<table>
<tr><td>17</td><td>8</td><td>16</td><td>12</td><td>24</td><td>19</td><td>23</td><td>11</td><td>20</td><td>13</td><td>4</td><td></td><td>19</td></tr>
<tr><td>24</td><td>23</td><td>20</td><td><u>19</u></td><td>17</td><td>8</td><td>16</td><td>12</td><td>11</td><td>13</td><td>4</td><td></td><td>23, 12</td></tr>
<tr><td>24</td><td><u>23</u></td><td>20</td><td><u>19</u></td><td>17</td><td>16</td><td>13</td><td><u>12</u></td><td>8</td><td>11</td><td>4</td><td></td><td>16, 11</td></tr>
<tr><td>24</td><td><u>23</u></td><td>20</td><td><u>19</u></td><td>17</td><td><u>16</u></td><td>13</td><td><u>12</u></td><td><u>11</u></td><td>8</td><td>4</td><td></td><td>4</td></tr>
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</table></td><td>M1
A1
A1ft
A1</td><td>1.1b
1.1b
1.1b
1.1b</td></tr>
<tr><td></td><td>e.g. Middle Left
<table>
<tr><td>17</td><td>8</td><td>16</td><td>12</td><td>24</td><td>19</td><td>23</td><td>11</td><td>20</td><td>13</td><td>4</td><td></td><td>19</td></tr>
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</table></td><td></td><td></td></tr>
<tr><td></td><td></td><td>(4)</td><td></td></tr>
<tr><td>(c)</td><td>Bin 1: 24 16
Bin 2: 23 17
Bin 3: 20 19
Bin 4: 13 12 11 4
Bin 5: 8</td><td>M1
A1</td><td>1.1b
1.1b</td></tr>
<tr><td></td><td></td><td>(2)</td><td></td></tr>
<tr><td colspan="4">(7 marks)</td></tr>
</table>

Notes for Question 1

(a)

B1: CAO – clear indication that the 16 was not placed in Bin 1 accept sight of $n < (17 + 8 + 16)$ or that the 4 was not placed in Bin 1 $n < (17 + 8 + 12 + 4)$ and that Bin 2 contains a total of 40.

(b)

M1: Quick sort, pivot, p, chosen (must be choosing middle left or right – choosing first/last item as the pivot is M0). After the first pass the list must read (values greater than the pivot), pivot, (values less that the pivot). **If only choosing one pivot per iteration then max of M1 only** – Bubble sort is not a MR and scores M0 If they make a slip with one value (one slip, one missing or one extra) at the start they can score M1 A0 A1ft A0

A1: First two passes correct and pivots chosen for third pass

A1ft: Third and fourth passes correct (follow through from their second pass and choice of pivots – if an error oversimplifies so that after 3 passes all sublists are of length 1 A0)

A1: CSO (correct solution only – all previous marks in this part **must** have been awarded)

SC: If the list is sorted in ascending order, then award a maximum of M1A1A0A0 (so 2 marks) as in the scheme above even if the list is re-ordered after the sort is complete

(c) NO MISREADS MARK EXACTLY TO THE SCHEME

M1: First 6 values placed correctly (the bold values) with at least 8 values placed in bins

A1: CSO so no additional/repeated values

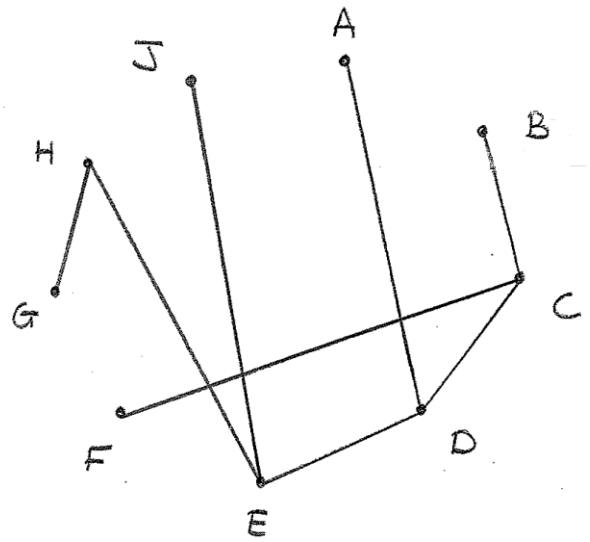
Sort ascending for reference

Middle Right

17	8	16	12	24	**19**	23	11	20	13	4		19
17	8	16	**12**	11	13	4	19	24	**23**	20		12, 23
8	**11**	4	12	17	**16**	13	19	20	23	24		11, 16
8	**4**	11	12	13	16	17	19	20	23	24		4
4	8	11	12	13	16	17	19	20	23	24		

Middle left

17	8	16	12	24	**19**	23	11	20	13	4		19
17	8	16	**12**	11	13	4	19	24	**23**	20		12, 23
8	**11**	4	12	17	**16**	13	19	20	23	24		11, 16
8	4	11	12	13	16	17	19	20	23	24		8
4	8	11	12	13	16	17	19	20	23	24		

Qn	Scheme	Marks	AOs
2. (a)	Prim's starting at D: DE, AD, EH; EJ, CD, BC; GH, CF	M1 A1 A1	1.1b 1.1b 1.1b
		(3)	
(b)	Weight of MST is 241 (miles)	B1	1.1b
		(1)	
(c)		B1	1.1b
		(1)	
(d)	482 (miles)	B1ft	1.1b
		(1)	
(e)	NNA starting at D: D – E – H – J – C – B – G – F – A – D 24 + 30 + 33 + 45 + 28 + 45 + 47 + 40 + 26 = 318 (miles)	M1 A1 A1	1.1b 1.1b 1.1b
		(3)	
(f)	The best upper bound is the one found in (e) as 318 is less than both 352 and 482	dB1ft	2.2a
		(1)	
(g)	(241 – 26) + 26 + 40 = 281 (miles)	M1 A1	3.1b 2.2a
		(2)	
(h)	The best lower bound is the one found in (g) as 281 is greater than 274	dB1ft	2.2a
		(1)	
			(13 marks)

Notes for Question 2

(a)

M1: First three arcs correctly chosen in order {DE, AD, EH, …} or first four nodes correctly chosen in order{D, E, A, H,…}. If any rejections seen at any point then **M1** (max) only. Order of nodes may be seen at the top of the matrix {3, -, -, 1, 2, -, -, 4, -} so please check the top of the matrix carefully. If they start at any other vertex M1 only for the first three arcs in order **(Starting at A AD DE EH)**.

A1: First six arcs correctly chosen in order {DE, AD, EH, EJ, CD, BC, …} or all nine nodes correctly chosen in order{D, E, A, H, J, C, B, G, F}. Order of nodes may be seen at the top of the matrix so for the first two marks accept {3, 7, 6, 1, 2, 9, 8, 4, 5} (do not condone any missing numbers e.g. the number 9 must be above F)

A1: CSO – all arcs correctly stated and chosen in the correct order. Candidates must be considering arcs for this final mark (do not accept a list of nodes or numbers across the top of the matrix unless the correct list of arcs (in the correct order) is also seen)

(b)

B1: CAO (241) – no units required (in this or any of the subsequent parts)

(c)

B1: Correct MST (If multiple attempts not rejected check all and award mark if any one is correct)

(d)

B1ft: Follow through double their answer to (b)

(e)

M1: Nearest neighbour route starting at D – must have at least D – E – H – J – C – B – … allow if stated in terms of arcs (DE EH HJ JC CB) but not for just numbering across the matrix

A1: CAO route (must return to D and can be stated as arcs DE EH HJ JC CB BG GF FA AD)

A1: CAO length (318)

(f)

dB1ft: Follow through their value from (e) (must include reason e.g. 318 is the lowest of the values found, but they do not need to mention the other values) **Must score at least M1 in (e)**

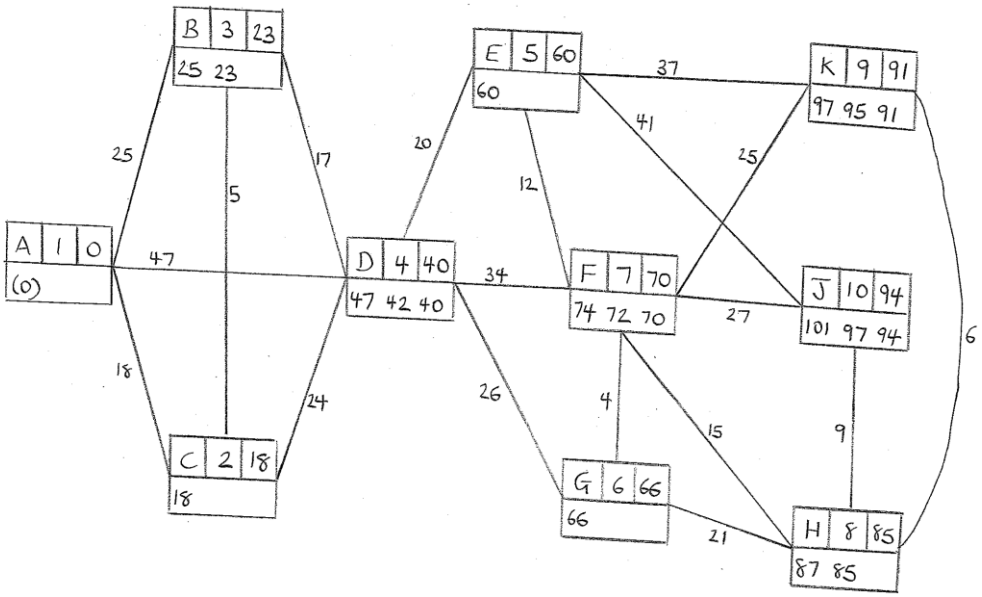
(g)

M1: (weight of their MST from (b) **or** 241 **only**) – 26 + 26(AD) + 40(AF) (oe so may not see the – 26 + 26). A correct answer of 281 can imply this (and the next) mark. Alternatively may delete A and find RMST (215) and add AD (26) and AF (40)

A1: 281

(h)

dB1ft: Follow through their "281" with 274 and makes conclusion (e.g. 281 is the bigger value, but they do not need to mention 274). **Must be a smaller value than their answer to (f) and must score at least M1 in (g)**

Qn	Scheme	Marks	AOs
3(a)		M1 A1 (ABC DE) A1 (GFH) A1ft (JK)	1.1b 1.1b 1.1b 1.1b
	Shortest path from A to J is ACBDGFHJ	A1	2.2a
		(5)	
(b)	If finishing at J then pair A, B, C and K: AB + CK = 23 + 73 = 96 AC + BK = 18 + 68 = 86* AK + BC = 91 + 5 = 96 If finishing at K then pair A, B, C and J: AB + CJ = 23 + 76 = 99 AC + BJ = 18 + 71 = 89 AJ + BC = 94 + 5 = 99	M1 A1ft M1 dep A1ft	3.1b 1.1b 1.1b 1.1b
	Finish at J and repeated AC, BD, DG, GF, FH, HK	A1	2.2a
	Total length of route is 413 + 86 = 499 (km)	A1ft	2.2a
		(6)	
(c)	e.g. Tarig's route length: (94 + 2) – 5 – 6 = 85 or with the addition of 413 giving 413 + 94 + 2 – 5 – 6 = 498 or 402 (413 – 5 – 6) + 96 (AJ) = 498 Therefore Tarig's route is shorter (dependent on 498 and 499 or 85 and 86 seen)	M1 A1	3.1b 2.2a
		(2)	
			(13 marks)

Notes for Question 3

In (a) it is important that all values at each node are checked very carefully – the order of the working values must be correct for the corresponding A mark to be awarded e.g. at B the working values must be 25 23 in that order (so 23 25 is incorrect)
It is also important that the order of labelling is checked carefully – some candidates start with a label of 0 at A (rather than 1) – which is fine. Also the order of labelling must be a strictly increasing sequence – so 1, 2, 3, 3, 4, … will be penalised once (see notes below) but 1, 2, 3, 5, 6, … is fine. Errors in the final values and working values are penalised before errors in the order of labelling

(a) M1: A larger value replaced by a smaller value in at least two of the working boxes at either B or D or F or H or K or J

A1: All values in A, B, C, D and E correct. Condone lack of 0 in A's working value
A1: All values G, F and H correct and the working values in the correct order. Penalise order of labelling only once per question (G, F and H must be labelled in that order and G must be labelled after A, B, C, D and E)

A1ft: All values in K and J correct on the follow through and the working values in the correct order. Penalise order of labelling only once per question. To follow through K, check that the working value at K follows from the candidate's final values from their feeds into K (which will come from nodes E, F and/or H (in the order in which the candidate has labelled them)) and that the final value, and order of labelling, follows through correctly. Repeat this process for J (which will possibly have working values from E, F and H with the order of these values determined by the candidate's order of labelling at E, F and H)

A1: CAO - correct path from A to J (ACBDGFHJ)

(b) Note: FT marks here are only for their values at K and J

M1: One correct set (either ABCK or ABCJ) of three distinct pairings of the correct four odd nodes (so must have AB + CK, AC + BK and AK + BC **or** AB + CJ, AC + BJ and AJ + BC)

A1ft: Any three rows correct including pairings **and** totals, from either set ABCK or set ABCJ (this can be 3 correct from one set or 2 correct from one set and 1 correct from the other set)

dM1: All six distinct pairings for nodes ABCK and ABCJ – dependent on first M mark

A1ft: All six rows correct including pairings **and** totals

A1: cao correct edges clearly stated and not just in their working. **Must** be edges AC, BD, DG, GF, FH, HK **and** clearly selecting to finish at J

A1ft: Either 499 from correct working or 413 + their "86" – **dependent on first four marks in this part**

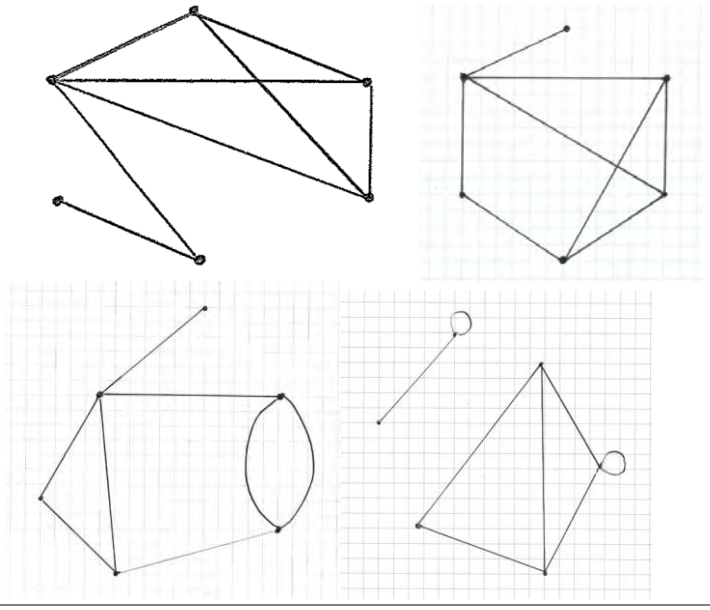
To check pairings use

AB	*CK*	96 or Final value at K + 5
AC	*BK*	86 or Final value at K - 5
AK	*BC*	96 or Final value at K + 5
AB	*CJ*	99 or Final value at J + 5
AC	*BJ*	89 or Final value at J - 5
AJ	*BC*	99 or Final value at J + 5

(c)

M1: Calculating the length of Tarig's route by considering the length of shortest path from A to J (either 94 **or** follow through final value at J from (a)) + 2 (as we can no longer take the shortest path from A to D as BC has been removed) – 5 (BC) – 6 (HK) A correct value of 498 or 85 implies this mark (but not from incorrect working)

A1: CAO (dependent on obtaining both correct values of 499 and 498 **or** 86 and 85)

Qn	Scheme	Marks	AOs
4. (a)	e.g. A graph cannot contain an odd number of odd vertices e.g. $\frac{1+2+3+4+5+6}{2}=10.5$ which is not an integer and so therefore not possible to have a graph with the given vertex orders	B1	1.2
		(1)	
(b)	e.g. T has at least one node of degree one or one node with odd degree	B1	2.4
		(1)	
(c)	6 nodes in T therefore the tree contains 5 arcs	B1	1.2
	$1+2+(4-x)+(2x-5)+(4x-11)+(3x-5)=2(5)$ $(8x-14=10)$	M1	3.1a
	$x=3$	A1	1.1b
	Alternatively $4-x>0 \Rightarrow x<4$ $\quad 4-x\geqslant 1 \Rightarrow x\leqslant 3$ $2x-5>0 \Rightarrow x>2.5$ $\quad 4x-11\geqslant 1 \Rightarrow x\geqslant 3$ $2.5<x<4 \Rightarrow x=3$ $\quad x=3$		
	Therefore, the degrees of the nodes are 1, 2, 1, 1, 1 and 4	M1	2.1
	T is not semi-Eulerian as there are more than two nodes of odd degree	A1	2.2a
		(5)	
(d)	e.g. 	B1	1.1b
		(1)	
			(8 marks)

Notes for Question 4

(a) In a and b condone poor language such as number of odd degrees instead of number of odd nodes

B1: CAO – common examples that score B1:

- **Cannot** have (a graph with an) **odd number** of **odd vertices**
- **Cannot** have a graph with **three odd vertices**
- The **sum of the degrees/order** (of the vertices) **is 21** which is **not even** therefore **not possible** (but not just for obtaining 21 and saying 'impossible'). The 21 must be linked either in words to the 'sum of the degrees/order' **or** explicitly showing 1+ 2 + 3 + 4 + 5 + 6 = 21 so just '21 is not even' scores B0
- The **sum of the degrees/order** (of the vertices) **is 21** which is **odd** therefore **not possible** (with equivalent justification of the 21 as in the previous bullet-point)
- $\frac{1+2+3+4+5+6}{2} = 10.5$ which is **not an integer** so therefore **impossible**. They do not have to explain that they are using the result that $\sum$ vertex degrees $= 2$ (no of arcs) but they must explain why a value of 10.5 leads to the required graph not being possible. A value of 10.5 with no working (or explanation) scores B0

(b) B1: CAO – correct reasoning that not all the vertices of T can have an even degree. Allow for this mark a general statement that no tree can be Eulerian as all trees contain at least two nodes of degree 1. Allow as a minimum statement that at least one of the nodes has a degree of 1 or at least one odd node (accept valency instead of degree or order)

(c)

B1: correctly using the fact that the number of arcs in T is 5 (possibly implied by later working – e.g. forming an equation for the sum of nodes = 2 x 5) An answer of $x = 3$ from correct working implies this mark.

Alternative approach either $4 - x > 0 \Rightarrow x < 4$ or $2x - 5 > 0 \Rightarrow x > 2.5$
May use $4x - 11 \geqslant 1 \Rightarrow x \geqslant 3$

M1: Forming an equation involving the degree of the six nodes **and** 2(5).

Alternative approach both inequalities stated and combined to obtain range of values for x

A1: CAO ($x = 3$) – must come from correct working
M1: Calculating the degree of the remaining vertices using their value of x. As a minimum we must see correct values for at least two more odd nodes. Must have an integer value for x (this can be implied by sight of 1, 2, 1, 1, 1, 4) Alternatively may clearly use $x = 3$ is odd and deduce that there are at least three odd nodes
A1: CAO (not Semi-Eulerian) – with correct reasoning

(d)

B1: CAO (any graph that is not isomorphic to G – must contain exactly 8 arcs) If a simple, connected graph the vertex with degree 1 must connect to either the vertex of degree 2 or degree 4. Note this does **not** have to be a simple graph and may **not** be connected
(Degrees are 1, 2, 3, 3, 3, 4)

<table>
<tr><th>Qn</th><th>Scheme</th><th>Marks</th><th>AOs</th></tr>
<tr><td>5.(a)</td><td>$x-y\geqslant 100\Rightarrow x-y-s_1+a_1=100$</td><td rowspan="3">B3,2,1</td><td>3.3</td></tr>
<tr><td></td><td>$x-5y\leqslant 0\Rightarrow x-5y+s_2=0$</td><td>3.3</td></tr>
<tr><td></td><td>$2x+3y\geqslant 350\Rightarrow 2x+3y-s_3+a_2=350$</td><td>3.3</td></tr>
<tr><td></td><td>$P=-x-y-M(a_1+a_2)$ and substitute expressions for a_1 and a_2
$\left(P+(1-3M)x+(1-2M)y+Ms_1+Ms_3=-450M\right)$</td><td>M1</td><td>2.1</td></tr>
<tr><td></td><td>e.g.
<table>
<tr><td>b.v</td><td>x</td><td>y</td><td>s_1</td><td>s_2</td><td>s_3</td><td>a_1</td><td>a_2</td><td>Value</td></tr>
<tr><td>a_1</td><td>1</td><td>-1</td><td>-1</td><td>0</td><td>0</td><td>1</td><td>0</td><td>100</td></tr>
<tr><td>s_2</td><td>1</td><td>-5</td><td>0</td><td>1</td><td>0</td><td>0</td><td>0</td><td>0</td></tr>
<tr><td>a_2</td><td>2</td><td>3</td><td>0</td><td>0</td><td>-1</td><td>0</td><td>1</td><td>350</td></tr>
<tr><td>P</td><td>$1-3M$</td><td>$1-2M$</td><td>M</td><td>0</td><td>M</td><td>0</td><td>0</td><td>$-450M$</td></tr>
</table></td><td>M1
A1</td><td>3.3
2.2a</td></tr>
<tr><td></td><td></td><td>(6)</td><td></td></tr>
<tr><td>(b)
(i)</td><td>$x=130$</td><td>B1</td><td>3.4</td></tr>
<tr><td>(ii)</td><td>When $x=130\Rightarrow y\leqslant 30, y\geqslant 26, y<130$ and $y\geqslant 30$</td><td>M1</td><td>3.1a</td></tr>
<tr><td></td><td>$y=30$</td><td>A1</td><td>2.2a</td></tr>
<tr><td></td><td></td><td>(3)</td><td></td></tr>
<tr><td colspan="4">(9 marks)</td></tr>
</table>

Notes for Question 5

(a) NOTE: if correct they must use one slack, two surplus and two artificial variables. Accept alternative letters for these as long as the artificial variables are clearly identifiable

B1: one correct equation or two correct inequalities (do not accept strict inequalities)

B1: two correct equations or three correct inequalities

B1: all three equations correct (please check suffices on s and a terms carefully – they may be in a different order)

M1: setting up the new objective which must be $P = -x - y - M(a_1 + a_2)$ and an attempt to substitute for their a_1 and a_2 (accept any equivalent form, which may not be fully simplified) (accept the use of Q instead of P throughout)

M1: setting up initial tableau – all four rows complete with two correct rows (but ignore b.v. column for this mark) **(Note the order of rows may be different from above) Check that the slack, surplus and artificial variables correspond to their equations**

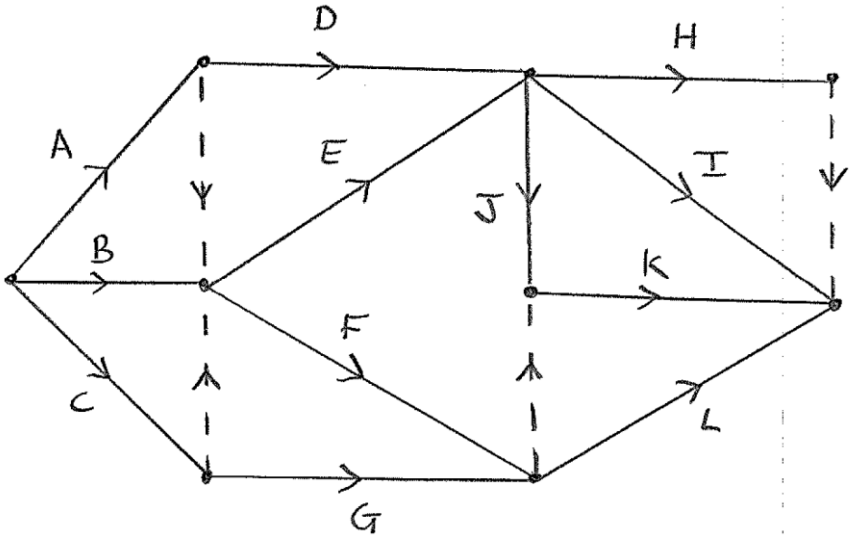
A1: CAO (any equivalent correct form, but the terms in the objective row must be simplified)

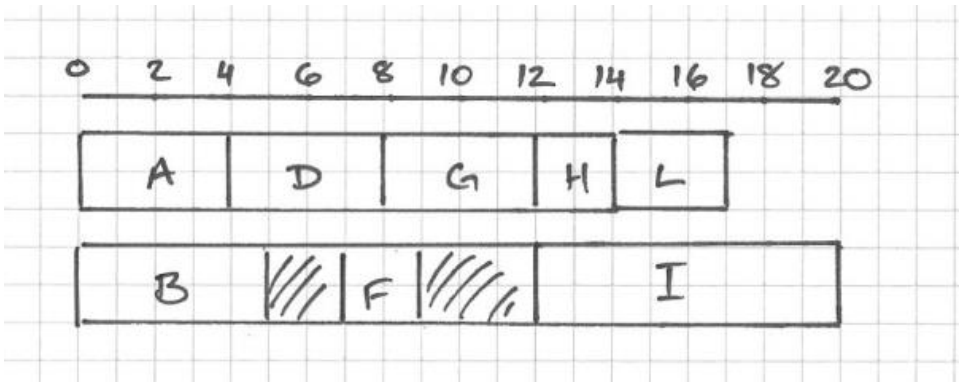
(b)

(i) B1: CAO ($x = 130$)

(ii) M1: Substitute $x = 130$ into candidate's inequalities from (a) (at least 3 inequalities seen or both $y \geqslant 30$ and $y \leqslant 30$) (condone $y \leqslant 130$)

A1: CAO ($y = 30$) we must see both $y \geqslant 30$ and $y \leqslant 30$ explicitly stated for this mark (must not follow from any incorrect working)

Question	Scheme	Marks	AOs
6. (a)	e.g,  	M1 A1 A1 A1 A1	2.1 1.1b 1.1b 1.1b 1.1b
		(5)	
(b)(i)	Critical path: C – E – J – K	B1	1.1b
(ii)	Minimum completion time: 20	B1	1.1b
(iii)	Total float on activity B is 2	B1	1.1b
(iv)	Total float on activity G is 4	B1	3.1b
		(4)	
(c)	e.g. (One worker continues to do the critical activities (in time 20)) Second worker completes A, D, G and H as originally planned but now completes activity L in the interval 14 to 20 (but must have started L by time 17 at the latest due to its duration of 3) Third worker completes B and F as planned but now completes activity I exactly in the time interval 12 to 20 so yes, the project can be completed by 3 workers	B2, 1, 0	2.4 2.4

	e.g.  Yes – can be completed by 3 workers		
		(2)	
			(11 marks)

For reference

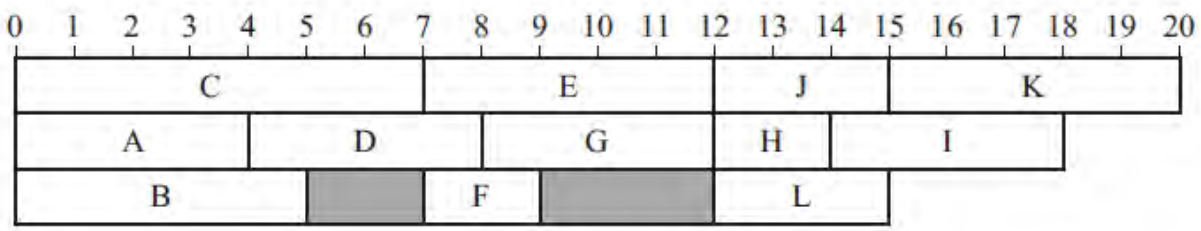


Figure 3

Notes for Question 6

Condone lack of, or incorrect, numbered events throughout. 'Dealt with correctly' means that the activity starts from the correct event but need not necessarily finishes at the correct event, e.g. 'H dealt with correctly' requires the correct precedences for this activity, i.e. D and E labelled correctly and leading into the same node and H starting from that node but do not consider the end event for H. **Activity on node is M0**

If an arc is not labelled, for example, if the arc for activity C is not labelled (but the arc is present) then this will lose the first A mark and the final (CSO) A mark – they can still earn the second A mark on the bod. If two or more arcs are not labelled then mark according to the scheme. Assume that a solid line is an activity which has not been labelled rather than a dummy (even if in the correct place for where a dummy should be)

Note: if they make multiple attempts which are not clearly replaced mark the one which is best for the candidate

Ignore incorrect or lack of arrows on the activities for the first four marks only

(a) M1: At least nine activities (labelled on arc), one start, at least two dummies placed

A1: Activities A, B, C, D and G dealt with correctly

A1: Activities E, F, H, I and J and the first two dummies (+ arrows) at the ends of A and C dealt with correctly (note H and I are interchangeable)

A1: Activities K, L and dummy at the end of F/G (+ arrow) dealt with correctly

A1: CSO – Final dummy (+ arrow) for uniqueness of H/I, all arrows present for every activity with one finish and no additional dummies. (Note direction of arrow on dummy between H and I is interchangeable) (Note the dummy could also be drawn at the start of H and I)

Please check all arcs carefully for arrows – if there are no arrows on dummies then M1A1max

Note that additional (but unnecessary) 'correct' dummies that still maintain precedence for the network should only be penalised with the final A mark if earned

(b)(i)

B1: CAO (critical path CEJK – in this order)

(ii)

B1: CAO (20)

(iii)

B1: CAO (2 – total float on B)

(iv)

B1: CAO (4 – total float on G)

(c)

B1: **Award this mark with an indication that either**

- activities I and L have swapped workers
- activity H has swapped worker and I now starts at 12

B1: dependent on first B mark – **must have a time reference for both I and either L or H for this mark (L or H may be a range)** completely correct reasoning including mention of workers, times and activities (for example, clear indication that (H and) L (in either order) must take place in the interval 12 to 20 **and** that I must now be done in the interval 12 to 20 or states starts at 12 but not just ends at 20) and either concludes **yes or clearly implies that it can be completed on time**

Note their explanation must not include an incorrect statement

Alternatively redraws the schedule for worker 2 and 3 with one completing H and L (in either order) in the time interval 12 – 20 and the other completing I **and** concludes that the project can be completed by 3 workers

<table>
<tr><th>Question</th><th>Scheme</th><th>Marks</th><th>AOs</th></tr>
<tr><td>7.(a)</td><td>The pivot for this first iteration came from the x-column</td><td>B1</td><td>1.1b</td></tr>
<tr><td></td><td>as it is now a basic variable</td><td>dB1</td><td>2.5</td></tr>
<tr><td></td><td></td><td>(2)</td><td></td></tr>
<tr><td>(b)</td><td>x-row: $4x+y-z\leqslant 40$ or s_3- row: $y+z\leqslant 26$</td><td>B1</td><td>3.4</td></tr>
<tr><td></td><td>Eliminating s_2 from the s_1 row using the x row:
$-\frac{1}{2}y+\frac{3}{2}z+s_1-2\left(10-x-\frac{1}{4}y+\frac{1}{4}z\right)=30$</td><td>M1</td><td>2.1</td></tr>
<tr><td></td><td>Eliminating s_2 from the P row using the x row:
$P-\frac{1}{4}y-\frac{11}{4}z+3\left(10-x-\frac{1}{4}y+\frac{1}{4}z\right)=30$</td><td>M1</td><td>1.1b</td></tr>
<tr><td></td><td>For either $P-3x-y-2z=0$ or $2x+z+s_1=50$</td><td>A1</td><td>1.1b</td></tr>
<tr><td></td><td>Alternatively
<table>
<tr><td>b.v.</td><td>x</td><td>y</td><td>z</td><td>s_1</td><td>s_2</td><td>s_3</td><td>Value</td></tr>
<tr><td>s_1</td><td>2</td><td>0</td><td>1</td><td>1</td><td>0</td><td>0</td><td>50</td></tr>
<tr><td>s_2</td><td>4</td><td>1</td><td>-1</td><td>0</td><td>1</td><td>0</td><td>40</td></tr>
<tr><td>s_3</td><td>0</td><td>1</td><td>1</td><td>0</td><td>0</td><td>1</td><td>26</td></tr>
<tr><td>P</td><td>-3</td><td>-1</td><td>-2</td><td>0</td><td>0</td><td>0</td><td>0</td></tr>
</table></td><td></td><td></td></tr>
<tr><td></td><td>LP:
(Maximise) $P=3x+y+2z$
Subject to: $2x+z\leqslant 50$
$4x+y-z\leqslant 40$
$y+z\leqslant 26$
$(x,y,z\geqslant 0)$</td><td>A1</td><td>2.2a</td></tr>
<tr><td></td><td></td><td>(5)</td><td></td></tr>
</table>

(c)

b.v.	x	y	z	s_1	s_2	s_3	Value	Row Ops
z	0	$-\frac{1}{3}$	1	$\frac{2}{3}$	$-\frac{1}{3}$	0	20	$\frac{2}{3}$r1
x	1	$\frac{1}{6}$	0	$\frac{1}{6}$	$\frac{1}{6}$	0	15	r2 + 0.25R1
s_3	0	$\frac{4}{3}$	0	$-\frac{2}{3}$	$\frac{1}{3}$	1	6	r3 – R1
P	0	$-\frac{7}{6}$	0	$\frac{11}{6}$	$-\frac{1}{6}$	0	85	r4 + 2.75R1

B1 1.1b
M1 2.1
A1 1.1b
B1 2.4

(4)

(d)(i) After the second iteration an optimal solution has not been found as the profit row still contains negative values — dB1 2.4

(ii) $z=20,\ x=15,\ s_3=6$ — dB1ft 2.2a

(2)

(e) e.g.

Their attempt is not correct as, if ***y*** is now a basic variable, then the ***y*** **column** should contain only **one value of 1** (in the third row) and therefore the entry of 1 in the profit row is incorrect (and should be 0) — B1 2.3

(1)

(14 marks)

Notes for Question 7

(a)

B1: CAO (correct column of x stated)

dB1: **Dependent on first B1** Correct statement that x is now a basic variable (oe) Accept that x now appears in the first (bv) column or states x column has one 1 and rest 0 or s_2 row has been replaced with x

(b)

B1: Either the constraint for the x-row **or** s_3**-** row correct (allow any equivalent form including non-integer coefficients but must be inequalities) Do not accept strict inequalities

M1: Eliminating s_2 from the equation from the s_1 row using the equation from the x row Correct constraint implies this mark

M1: Eliminating s_2 from the equation from the P row using the equation from the x row Correct objective implies this mark

A1: $P-3x-y-2z=0$ **or** $2x+z+s_1=50$ (allow any equivalent form including non-integer coefficients but must have been simplified to a single term in each variable) Correct constraint or objective implies the corresponding M mark

A1: Correct LP formulation (condone lack of 'maximise' and condone lack of the non-negative trivial constraints)

Note: It is possible to score M1 M0 A1 A0 or M0 M1 A1 A0

Alternatively – reproduces original tableau

B1: s_2 row correct

M1: s_1 row correct - Correct constraint implies this mark

M1: P row correct - Correct objective implies this mark

A1: fully correct tableau

A1: Correct LP formulation (condone lack of 'maximise' and condone lack of the non-negative trivial constraints)

(c) Note – accept correct recurring decimals in place of fractions

B1: Pivot row completely correct including change of b.v.

M1: All **values** in one of the non-pivot rows correct (so ignore b.v. column and 'Row Ops' column) **or** one of the 'non zero and one' columns (which are y, s_1, s_2 or Value) correct (must have pivoted on the correct value)

A1: cao **all** values including b.v. column – ignore 'Row Ops' column for this mark

B1: Correct row operations stated (allow alternative numbering of rows as long as this is clear. Condone use of R_1 throughout)

(d)(i)

dB1: CAO (profit row still contains negative values (but not negative variables) – dependent on the M mark in (c) and a completed profit row in (c) (accept P row or objective row but not operations row or bottom row) Do not accept that not all values are positive

dB1ft: Follow through their values of z, x, and s_3 – dependent on the M mark in (c) and a completed tableau in (c) ignore mention of any other variables including P **All values must be positive**

(e)

B1: CAO (e.g. correct indication that the (basic variable) column for y is not correct – see bold statement for the minimum acceptable) Accept there should not be a 1 in the objective row in the y column or that there should not be two 1s in the y column. If s_2 column also mentioned then B0

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